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Math 350 Winter 2016

Test #1

You are being evaluated on the presentation, as well as the correctness, of your answers. Try to answer questions in a clear, direct, and efficient way. Show your work and simplify your solutions. Where appropriate, your solutions should include definitions and references to theorems.

[6] **1.**(a) Write $z = -2\sqrt{3} - 2i$ in polar form.

(b) Evaluate $(-1-i)^9$.

(c) Find all distinct values of $(8i)^{1/3}$.

(d) Evaluate $\sin(\frac{\pi i}{2})$.

[4] **2.** Describe the set of points z in the complex plane that satisfy the following equations and determine which of these set is a domain.

(a) $(\text{Re } z)^2 > 1$

(b) $\text{Re}(\bar{z} - i) = 2$

[3] **3.** Use the formal definition of limits to show that the function $f(z) = \frac{iz}{3}$ is continuous at $z = i$.

[3] **4.** Determine whether $f(z)$ is continuous at $z = 0$.

$$f(z) = \begin{cases} 0 & z = 0, \\ \frac{\text{Im } z}{z} & z \neq 0. \end{cases}$$

[4] **5.** State the definition of an analytic function on a set S and show that $z\bar{z}$ is nowhere analytic.

[5] **6.** Consider the function $v(x, y) = \sin x \cosh y$.

(a) Show that $v(x, y)$ is harmonic.

(b) Use part (a) to find an analytic function $f(z) = u(x, y) + iv(x, y)$ that maps the origin to the point $(1, 0)$.